**Instructions**: Complete each of the following exercises for practice.

1. Sketch the curve $\mathbf{r}(t)$ in $\mathbb{R}^2$, compute $\mathbf{r}'(t)$, and sketch both $\mathbf{r}(a)$ and the tangent vector $\mathbf{r}'(a)$.

   (a) $\mathbf{r}(t) = \langle t^2, t^3 \rangle$, $a = 1$ $\mathbf{r}(t) = e^{2t}\mathbf{i} + 2t\mathbf{j}$, $a = 0$

   (b) $\mathbf{r}(t) = \langle \cos(t) + 1, \sin(t) - 1 \rangle$, $a = -\frac{\pi}{3}$

2. Compute the derivative of the vector function.

   (a) $\mathbf{r}(t) = \left\langle \sqrt{t-2}, 3, t^{-2} \right\rangle$

   (b) $\mathbf{r}(t) = t^2\mathbf{i} + \cos(t^2)\mathbf{j} + \sin^2(t)\mathbf{k}$

3. Find the unit tangent vector $\mathbf{T}(t)$ for $\mathbf{r}(t)$ at $t = a$.

   (a) $\mathbf{r}(t) = \left\langle t^2 - 2t, 1 + 3t, \frac{1}{3}t^3 + \frac{1}{2}t^2 \right\rangle$, $a = 2$

   (b) $\mathbf{r}(t) = \cos(t)\mathbf{i} + 3t\mathbf{j} + 2\sin(2t)\mathbf{k}$, $a = 0$

   (c) $\mathbf{r}(t) = \langle \arctan(t), 2e^{2t}, 8te^t \rangle$, $a = 0$

   (d) $\mathbf{r}(t) = \sin^2(t)\mathbf{i} + \cos^2(t)\mathbf{j} + \tan^2(t)\mathbf{k}$, $a = \frac{\pi}{4}$

4. For $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ compute $\mathbf{r}'(t)$, $\mathbf{T}(t)$, $\mathbf{r}''(t)$, $\mathbf{r}' \cdot \mathbf{r}''(t)$, and $\mathbf{r}' \times \mathbf{r}''(t)$.

5. Compute parametric equations of the tangent line to curve $x = t^2 + 1$, $y = 4\sqrt{t}$, $z = e^{t^2 - t}$ at $(2, 4, 1)$.

6. Find the point on $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), e^2 \rangle$ for $0 \le t \le \pi$ with tangent line parallel to plane $\sqrt{3}x + y = 1$.

7. Compute $\displaystyle\int_{t=0}^{1} \left( \frac{1}{t+1}\mathbf{i} + \frac{1}{t^2+1}\mathbf{j} + \frac{1}{t^2+1}\mathbf{k} \right) dt$.

8. Compute $\mathbf{r}(t)$ supposing $\mathbf{r}'(t) = \langle t, e^t, te^t \rangle$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

9. Prove that $\dfrac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}(t) \times \mathbf{r}''(t)$ and $|\mathbf{r}(t)| \dfrac{d}{dt}\left[\mathbf{r}(t)\right] = \mathbf{r}(t) \cdot \mathbf{r}'(t)$ for all vector functions $\mathbf{r}(t)$.

10. Suppose curve $\mathbf{r}(t)$ is perpendicular to its tangent $\mathbf{r}'(t)$ for all $t$. Prove $\mathbf{r}(t)$ is on a sphere about the origin.